# ABSTRACTS

OF

**SYMPOSIUM ON RIEMANNIAN-FINSLER GEOMETRY**

IN THE CONFERENCE OF

RAMANUJAN MATHEMATICAL SOCIETY

AT

IISER, PUNE

(23-27 JUNE, 2014)

## CONTENTS

## 1. **Title: Riemannian-Finsler Geometry in the Large**

K.Shiohama
Department of Applied Mathematics
Fukuoka Institute of Technology
Fukuoka, 841–0108, JAPAN

1.1. **Introduction.** We are interested in the global Finsler geometry. Riemannian geometry in the large has been discussed by many people since the beginning of the last century. In particular, curvature and topology of complete Riemannian manifolds is one of the main stream of differential geometry. For the development of Finsler geometry in the large, we discuss global Riemannian geometry in many aspects. The key point is the anti-symmetric property of distance function on a Finsler manifold. We disciss global results in Riemannian geometry which can be developed in Finsler geometry. Simple extension of Riemannian results to Finsler geometry is not interesting. There are unknown phenomena which we have found in due course of the Finsler extensions of Riemannian results.

We discuss convex functions and strictly convex functions on complete Finsler manifolds. We then discuss the Finsler extension of the Rauch conjecture on the cut locus and conjugate locus in the tangent space. Preceding results in Riemannian geometry are seen in [4], [6], [8], [12] and [14]. The Finsler results are reffered to [10], [11].

The completion of our work was facilitated by a fellowship of the program "Exploratory Exchange under the Japan-India Cooperative Science Program between JSPS and DST (2014)" to the author for a scientific visit to Varnasi, INDIA. I would like to express my thanks to this organization and also to Professor Ravi Kulkarni for kindly opening the Finsler geometry session at the Ramanujan Mathematical Society Meeting.

1.2. **Preliminaries.** The definitions and notations used in our global Finsler geometry and the Basic facts will be stated. For a smooth $n$-manifold, $n \geq 2$, the fundamental function $F : TM \to [0, \infty)$ over the tangent bundle $TM$ of $M$ which has the following properties:

(1) $F$ is smooth on $TM \setminus 0$, (regularity).
(2) $F(x, \lambda y) = \lambda F(x, y)$, for all $\lambda > 0$ and for all $(x, y) \in TM$ (positive homogeneity).
(3) The Hessian matrix $(g_{ij})$ is positive definite for all $(x, y) \in TM \setminus 0$, (strong convexity).

Here $g_{ij}$ is defined by

$$g_{ij} = \frac{1}{2}\frac{\partial^2 F}{\partial y^i \partial y^j}$$

The length $L(c)$ of a curve $c : I \to (M, F)$ defined on an interval $I$ is given as

$$L(c) := \int_I F(c(t), \dot{c}(t))\, dt.$$

The intrinsic distance function on $(M, F)$ is defined by

$$d(p, q) := \inf \{L(c) | c \quad \text{is a curve joining } p \text{ to } q\}.$$

Therefore the positive homogeneity of $F$ implies that $L(c)$ is not necessaryily equal to that of its reversed curve $c^{-1}$, and hence we have

$$d(p, q) \neq d(q, p), \quad p, q \in M$$

1.3. **Geodesics, Cut locus, Conjugate locus and (forward) Poles.** A geodesic $\gamma : [a, b] \to (M, F)$ on $(M, F)$ is defined by the nonlinear second order ordinary differential equation. For an arbitrary given vector $(p, u) \in TM$, there is a unique solution of a geodesic $\gamma_u : [0, \varepsilon) \to M$ with the initial condition $\gamma_u(0) = p$, $\dot{\gamma}_u(0) = u$. The exponential map $\exp_p : M_p \to M$ is defined by

$$\exp_p u := \gamma_u(1), \quad \text{wherever it makes sense.}$$

Clearly, $\exp_p$ restricted to a small metric ball centered at the origin is an embedding into $M$. Let $\Sigma_p \subset M_p$ for a point $p \in M$ be the indicatrix;

$$\Sigma_p := \{u \in M_p \,|\, F(p, u) = 1\}$$

Define a function $s : \Sigma_p \to \mathbb{R}$ such that if $u \in \Sigma_p$ and if $\gamma_u(t) := \exp_p tu$, $t \geq 0$, then

$$s(u) := \sup\{t > 0 \,|\, d(p, \gamma_u(t)) = t\}.$$

We say that $\gamma_u : [0, \infty) \to M$ is a *(forward) ray* if $s(u) = \infty$, and $s(u)u \in M_p$ is a *tangential cut point to $p$ along $\gamma_u$* if $s(u) < \infty$. Let $U_p \subset M_p$ be the domain such that $U_p$ is star-shaped with respect to the origin of $M_p$ and $U_p := \{tu \,|\, 0 \leq t < s(u), u \in \Sigma_p\}$. The $U_p$ is the maximal domain on which $\exp_p | U_p : U_p \to M$ is an embedding. The boundary $\widetilde{C}(p) := \partial U_p$ of $U_p$ is called the *tangential cut locus to $p$* and $C(p) := \exp_p \widetilde{C}(p)$ is the *cut locus to $p$*. A point $p \in M$ is called a *(forward) pole* if and only if $C(p) = \emptyset$ (or $U_p = M_p$). If $\gamma_u : [0, \infty) \to M$ is a geodesic and if $s(u) < \infty$, then $\gamma_u(s(u))$ is called the *cut point to $p$ along $\gamma_u$*. A cut point $\gamma_u(s(u))$ to $p$ along $\gamma_u$ has the property that either $\gamma_u(s(u))$ is joined to $p$ by another minimizing geodesic or it is conjugate to $p$ along $\gamma_u$). A point $\gamma_u(c(u))$ is called *the first conjugate point to $p$ along $\gamma_u : [0, \infty)$* iff

$$c(u) := \sup\{t > 0 \,|\, \det(d\exp_p)_{su}) \neq 0 \quad \text{for all } s \in [0,t)\}.$$

The tangential conjugate locus $\widetilde{J}(p) \in M_p$ and conjugate locus $J(p)$ to $p$ are given as:

$$\widetilde{J}(p) := \{c(u)u \,|\, u \in \Sigma_p\}, \quad J(p) := \exp_p(\widetilde{J}(p)).$$

The injectivity radius function $i : M \to \mathbf{R}$ is defined to be

$$i(p) := \inf\{s_u | u \in \Sigma_p\}.$$

1.4. **Completeness.** We say that $(M, F)$ is *(forward) complete* if there is a point $p \in M$ at which the exponential map is well defined over the whole tangent space $M_p$. A classical well known Whitehead theorem states that every point $p \in (M, F)$ admits a metric $r(p)$-ball $B(p, r(p)) := \{x \in M \,|\, d(p, x) < r(p)\}$ centered at $p$ which is *convex* in the following sense: If $x, y \in B(p, r(p))$, then a minimizing geodesic joining $x$ to $y$ is contained in $B(p, r(p))$.

1.5. **Convex functions.** A function $\varphi : (M, F) \to \mathbb{R}$ is called *convex* iff $\varphi \circ \gamma : [a, b] \to \mathbb{R}$ is convex along every geodesic $\gamma : [a, b] \to (M, F)$, i.e.

$$\varphi \circ \gamma((1-\lambda)s + \lambda t) \leq (1-\lambda)\varphi \circ \gamma(s) + \lambda\varphi \circ \gamma(t),$$

for all $a \leq s < t \leq b$ and $0 \leq \lambda \leq 1$. A convex function on $(M, F)$ is called *strict* iff the above inequality is strict for all $a \leq s < t \leq b$ and $0 \leq \lambda \leq 1$.

1.6. **review of known facts.** We introduce the Riemannian results which are extended to Finsler manifolds in [10] and [11] and [14]. Throughout this section let $(M, g)$ be a complete Riemannian $n$-manifold. We refer the basic facts of Riemannian geometry to [2].

**Theorem 1.1** (see [4])**.** If a locally non-constant convex function $\varphi : (M, g) \to \mathbb{R}$ has a disconnect level, we then have:

(1) the infimum $\lambda := \inf_M \varphi$ of $\varphi$ is attained at a point $p \in M$;
(2) the minimum set $\varphi^{-1}(\{\lambda\})$ is a connected, complete, totally geodesic hyper surface without boundary, and its normal bundle is trivial;
(3) all the levels of $\varphi$ are homeromrphic to $\varphi^{-1}(\{\lambda\})$

**Theorem 1.2** (see [5])**.** If a locally non-constant convex function $\varphi : (M, g) \to \mathbb{R}$ has no minimum, then there exists differentiable manifold $N$ such that $M$ is diffeomorphic to $N \times \mathbb{R}$.

Some of the above theorems have the Finsler extension and are stated in Sabau's talk. In connection with the strict convexity we have:

**Theorem 1.3** (see [3])**.** Let $(M, g)$ be a complete noncompact Riemannian $n$-manifold with positive sectional curvature. Then the exponential map at every point is proper.

**Theorem 1.4** (see Theorem A in [14])**.** If $(M, g)$ admits a strictly convex exhaustion function, then every compact subgroup of the isometry group $I(M)$ of $(M, g)$ has a common fixed point.

**Theorem 1.5** (see Theorem B in [14])**.** If $(M, g)$ admits a proper strictly convex function without minimum, then $I(M, g)$ is compact.

Some to these theorems are extended to Finsler manifolds and will be stated in Tiwari's talk.

1.7. **The Rauch conjecture and poles.** The classical Rauch conjecture states that if $(M, g)$ is compact and simply connected, then $\widetilde{C}(p) \cap \widetilde{J}(p) \neq \emptyset$ holds at each point $p \in (M, g)$. The Rauch conjecture is true for every Riemannian 2-manifolds homeomorphic to $S^2$ and for every compact rank one symmetric space of compact type. However this conjecture was negatively solved by Weinstein in [12] by proving that if $M$ is not homeomorphic to $S^2$, then there exists a metric $g$ and a point $p \in M$ such that the Rauch conjecture is not valid at that point. Recently, we have discussed the Rauch conjecture [6] in connection with the cut locus $C(p) \subset M$ to a point $p \in (M, g)$. More precisely, we have proved that if a complete Riemannian $n$-manifold $(M, g)$ admits a point $p \in M$ with $C(p) \neq \emptyset$ and if the Rauch conjecture does not hold at $p$, then there exists a non-minimal geodesic $\sigma$ joining $p$ to every point $q \in M \setminus C(p)$. Moreover, $\sigma$ has the property that its length $L(\sigma)$ is not longer than any other non-minimal geodesics in the non-degenerate loop space $\Omega_{pq}$ joining $p$ to $q$.

We discuss the Riemannian resuts obtained in [6] on complete Finsler $n$-manifolds.

**Theorem 1.6** (see Theorem 1 in [6])**.** Let $p \in (M, g)$ be a point such that $C(p) \neq \emptyset$. We then have:

(1) The Rauch conjecture holds at $p$. Namely, $\widetilde{C}(p) \cap \widetilde{J}(p) \neq \emptyset$.
(2) There are at least two geodesics joining to to every point $q \in M \setminus \{p\}$, if the Rauch conjecture does not hold at $p$. In particular, $p$ is the base point of a non-trivial geodesic loop if $q = p$.
(3) If $q \in (M, g)$ is a pole and if $x \in (M, g)$ is not a pole, then the Rauch conjecture holds at $x$, i.e. $\widetilde{C}(x) \cap \widetilde{J}(x) \neq \emptyset$.

**Proposition 1.1.** Let $p \in (M, F)$ be a backward pole. We then have either the Rauch conjecture holds at a point $q \in (M, F)$, or otherwise $q$ is a forward pole.

**Remark 1.** If $(M, g)$ is a complete Riemannian manifold with a pole at $p \in M$, then the Rauch conjecture is valid at every point $q \in M$. If $(M, F)$ is a complete finsler manifold with a backward pole $p \in (M, F)$, we then have the same conclusion. Namely, the Rauch conjecture holds at every point $q \in (M, F)$ with $C(q) \neq \emptyset$.

For given two distinct points $p, q \in (M, F)$ and for a positive number $\ell > d(p, q)$, we denote by $E_{p,q}(\ell)$ the ellipse with foci at $p$ and $q$ and with radius $\ell$, and by $B_{p,q}(\ell)$ its interior :

$$E_{p,q}(\ell) := \{x \in M \mid d(p, x) + d(x, q) = \ell\},$$
$$B_{p,q}(\ell) := \{x \in M \mid d(p, x) + d(x, q) < \ell.$$

With this notation we have:

**Theorem 1.7** (compare Lemma 6 ; [6])**.** Assume that the Rauch conjecture does not hold at a point $p \in (M, F)$ and that $C(p) \neq \emptyset$. Let $q \in (M, F) \setminus C(p)$. Assume that $x_0 \in C(p)$ satisfies $x_0 \in E_{pq}(\ell) \cap C(p)$ and $B_{pq}(\ell) \cap C(p) = \emptyset$. Here we set $\ell := d(p, x_0) + d(x_0, q)$.

(1) Then there exists a geodesic $\gamma_{pq} : [0, \ell] \to (M, F)$ with $\gamma_{pq}(0) = p, \gamma_{pq}(d(p, x_0)) = x_0$ and $\gamma_{pq}(\ell) = q$ such that it is not minimizing and such that if $\sigma$ is a non-minimizing geodesic belonging to $\Omega_{pq}$, then $L(\sigma) \geq L(\gamma_{pq}) = \ell$.

(2) We have exactly two minimizing geodesics, say, $T_1(p, x_0)$ and $T_2(p, x_0)$ joining $p$ to $x_0$.

**Theorem 1.8.** Let $(M, F)$ be a compact Finsler $n$-manifold. If the Rauch conjecture does not hold at a point $p \in (M, F)$, then there exists a geodesic loop $\gamma_p : [0, 2\ell_p] \to (M, F)$ with $\gamma_p(0) = \gamma_p(2\ell_p) = p$ such that $i(p) \leq \ell_p$ and $\gamma_p(\ell_p) \in C(p)$. Namely, both $\gamma_p|_{[\ell_p, 2\ell_p]}$ and its reversed curve $\gamma_p|^{-1}_{[\ell_p, 2\ell_p]}$ are minimizing geodesics.

## References


[1] D.Bao–S.S.Chern–Z.Shen, An Introduction to Riemannian-Finsler Geometry, **200**, Graduate Texts in Math. Springer-Verlag, New York, 2000.

[2] J.Cheeger–D.Ebin, Comparison Theorems in Riemannain Geometry, AMS Chelsia Publ. Amer. Math. Soc. Provdence, Rhode Island, 2008.

[3] D.Gromoll–W.Meyer, *Completeopen manifolds of positive curvature,* Ann. of Math. **90**, 75–90(1969) .

[4] R.Greene–K.Shiohama, *Convex functions on compete noncompact manifolds; Topological structure,* Invent. Math. **63**, 129–157(1981).

[5] R.Greene–K.Shiohama, *Conve functions on complete noncompact maniflds: Differentiable structure,* Ann.Scient. Ëc. Norm. Sup. **14**, 357–367, (1981).

[6] N.Innami–K.Shiohama–T.Soga, *The cut loci, conjugate loci and poles in a complete Riemannian manifold*, GAFA, **22**, 1400–1406 (2012).

[7] W.Klingenberg, Riemannian Geometry, de Gruter Studies in Mathematics 1, Walter de Gruyter, Berlin–New York, 1982.

[8] H.Rauch, Geodesics and Curvature in Differential Geometry in the Large, Yeshiva University Press, New York (1959).

[9]

[10] S.Sabau–K.Shiohama, *Topology of complete Finsler manifolds admitting convex functions.* Preprint, 2013.

[11] B.Tiwari–K.Shiohama, *Strictly convex functions and isometry groups of complete Finsler manifolds,* Preprint, 2014.

[12] A.Weinstein, *The cut locus and conjugate locus of a riemannian manifold*, Ann. of Math. **87**, 29–41 (1968).

[13] J.H.C.Whitehead, *On the covering of a complete space by the geodesics through a point*, Ann. of Math. **36**, No.3, 679–704 (1935).

[14] T.Yamaguchi, *The isometry groups of Riemannian manifolds admitting strictly convex functions,* Ann. Scient. Ëc. Norm. Sup. **15**, 205–212 (1982).

2. **Title: Remarks on the Geometry of Geodesics on Finsler manifolds**

Sorin Sabau
Tokai University, Sapporo
JAPAN

The geometry of geodesics is one of the main research topics in Differential Geometry. Many fundamental results concerning the Geometry and topology of Riemannian manifolds were derived in this way. On the other hand, such research is not so frequent in classical Finsler geometry.

However, in the last years, a visible progress in this direction can be witnessed. Many classical results from Riemannian geometry were extended in the realm of Finsler geometry by K. Shiohama, M. Tanaka, S. Ohta, K. Kondo and others.

In the present talk I will describe the importance of the Geometry of Geodesics on a Finslerian manifolds and introduce some new results obtained by this method. In special the notion of cut locus of a Finsler manifold and global properties of geodesics are involved. Moreover, we will show that the existence of a convex function on a Finsler manifold impose topological restrictions on the base manifold (joint research with Professor K. Shiohama).

3. **Title: Some topics in conformal Finsler geometry**

Tadashi Aikou
Kagoshima University
JAPAN

In this talk, I will report on some topics in Finsler geometry. In particular, I will define the invariant curvature tensor by one-sided projective change of Finsler connections. I will show a characterization of conformally flat Finsler manifolds by using the invariant curvature tensor. This research is a joint work with Mr. Jenizon who is a graduate student at Kagoshima University.

## 4. **Title:Quasi -metric structures associated to Finsler metrics**

Hideo Shimada
Tokai University, Sapporo Campus
JAPAN

We investigated the relation between weighted quasi-metric spaces and Finsler spaces. We show the induced metric of a Finsler metric with reversible geodesics is a weighted quasi-metric. Weighted quasi-metric spaces were initially introduced in the context of theoretical computer science. And their topological properties were investigated. In the paper [1], we show the metric structure induced from Finsler metric with reversible geodesics is actually a weighted quasi-metric. The main result is the following: Theorem 1. Let M be an n-dimensional simply connected smooth manifolds. A Finsler metric F induced a generalized weighted quasi-distance on M if and only if it is a Randers change of an absolute homogeneous Finsler space by an exact one-form beta. To prove this theorem, we used our Theorem 2 [2, 3]. Let $(M, F)$ be Finsler space whose fundamental function is obtained by a Randers change of an absolute homogeneous Finsler metric by a 1-form beta. Then $(M, F)$ is with reversible geodesics if and only if the one form beta is closed. This is a joint work with S.V. Sabau and K. Shibuya [1].

**References**

[1]Sorin V. Sabau, Kazuhiro Shibuya and Hideo Shimada, Metric structures associated to Finsler metrics, to appear in Publ. Math. Debrecen, (2014).
[2] Ioana Masca, Sorin V. Sabau and Hideo Shimada, Reversible geodesics for $(\alpha, \beta)$-metrics, Intl. J. Math. 21 (2010), 1071-1094.
[3] Ioana Masca, Sorin V. Sabau and Hideo Shimada, Two dimensional $(\alpha, \beta)$-metrics with reversible geodesics, Publ. Math. Debrecen, 82/2 (2013), 485-501.
[4] Paulo Vitolo, The representaion of weighted quasi-metric spaces, Rend. Instit. Math. Univ. Trieste, Vol. XXX1(l995), 95-100.

5. **Title: Some Remarks on Geodesics of Finsler Spaces**

Tetsuya Nagano
Nagasaki, Sieboldt
JAPAN

The notion of reversibility of geodesics of Finsler spaces was defined by M.Crampin in 2003. T.Nagano defined the notion of linear parallel displacements of curves in Finsler spaces in 2008. In my talk, speaker will state a relation of strictly reversible and lpd-symmetric for geodesics and some properties for lpd-symmetric curves.

6. **Title:Grove-Shiohama type sphere theorem in Finsler geometry**

Kei Kondo
Yamaguchi University,
JAPAN

In this talk I will introduce several sphere theorems of Grove-Shiohama type for a certain class of compact Finsler manifolds and other related results.

7. **Title: Curvature inherited symmetries in Finsler spaces**

P. N. Pandey
University of Allahabad, Allahabad
INDIA

## 8. Title: Time Measure on Slope

H.S.Shukla
Dean, Faculty of Science,
DDU Gorakhpu University, Gorakhpur-273009, India

**Abstract:** Suppose that we have an orthonormal coordinate system $(x, y, z)$ in an ordinary space; the $(x, y)$-plane is the sea level, $z(\geq 0)$ shows the altitude above sea level, and a slope of a mountain is regarded as the graph $S$ of a smooth function $z = f(x, y)$ of two arguments. If a person is able to walk $v$ meters per minute on a horizontal plane, he can walk the distance $vt+(\frac{gt^2}{2})\sin\epsilon\cos\theta$in $t$ minutes along a straight road at an angle $\theta$ with the direct downhill road on a plane inclined at an angle $\epsilon$ to the horizontal plane. Thus the distance walked in unit time is $r = v+ a\cos\theta$, where $a = (g/2)\sin\epsilon$. Hence with respect to the time measure, a plane with an angle $\epsilon$ of inclination can be regarded as a Minkowski plane, whose indicatrix curve is a limacon. Applying Okubos method the fundamental function $L(x, y; x, y)$ is derived as $L(x, y, \dot{x}, \dot{y}) = \frac{\alpha^2}{v\beta - w\alpha}$ ,$w = g/2$, where $\alpha^2 = \dot{x}^2 + \dot{y}^2 + (\dot{x}f_x + \dot{y}f_y)^2$, $\beta = \dot{x}f_x + \dot{y}f_y$. By normalization, we get $L = \frac{\alpha^2}{\alpha-\beta}$, which is Matsumoto metric. We show that a two dimensional differential manifold $M$ endowed with the above fundamental function is a Finsler manifold if and only if $f_x^2+f_y^2 \leq \frac{v^2}{g^2-v^2}$.

## 9. Title: Model preserving conformal transformations of Finsler spaces

B. N. Prasad
C-10 Surajkund Colony, Gorakhpur

**Abstract:** In the present lecture we will discuss the concept of modal preserving conformal transformations of Finsler spaces. Various modals of Finsler spaces have been introduced in literature. Riemannian space, Berwald space, Landsberg space, Douglas space, C-reducible space, Semi C-reducible space P-reducible space and spaces with $(\alpha, \beta)$- metric are some important Finsler models. We will discuss that what type of conformal transformation will transform these models to the models of same type.

## 10. **Title: On Bounded Cartan Torsion of Special Finsler $(\alpha, \beta)$-Metrics**

S.K. Narasimhamurthy
Department of P.G. Studies and Research in Mathematics,
Kuvempu University, Shankaraghatta - 577451,
Shimoga, Karnataka, INDIA.
E-mail: nmurthysk@hotmail.com

**Abstract:** In this paper, we proved that some special Finsler $(\alpha, \beta)$-metrics have bounded Cartan torsion. Further more, we find relation between the norm of Cartan and mean Cartan torsion for the class of $(\alpha, \beta)$-metrics.
**Key Words:** Finsler Space, $(\alpha, \beta)$-metrics, Norm of Cartan, Bounded Cartan Torsion, Mean Cartan Torsion.

### References

[1] E.Cartan, *Les espaces de Finsler, .* Actualities 79, Paris, 1934.

[2] Z. Shen, *On Finsler geometry of submanifolds.* Math. Ann. 311(3)(1998), 549-576.

[3] D. Bao, S.S. Chern and Z. Shen, *An Introduction to Riemann-Finsler Geometry,* Springer, 2000.

[4] A. Tayebi and H. Sadeghi, *On Cartan torsion of Finsler metrics,* Publ. Math. Debrecen. 82(2013), accepted.

[5] M. Matsumoto, *On C-reducible Finsler spaces,* Tensor. 24(1972), 29-37.

[6] Z. Shen, *On Finsler geometry of submanifolds,* Math. Ann. 311(3)(1998), 549-576.

[7] M. Matsumoto, *Theory of Finsler spaces with* $(\alpha, \beta)-$*metric,* Rep. Math. Phys. 31(1992), 43-84.

[8] Z. Shen, *Differential Geometry of Spray and Finsler Spaces,* Kluwer Academic Publishers, 2001.

[9] X. Mo and L. Zhou, *A class of Finsler metrics with bounded Cartan torsion,* Canad. Math. Bull. 53(2010), 122-132.

## 11. Title: Finsler space subjected to a Kropina change with an *h*-vector

M. K. Gupta
Department of Pure & Applied Mathematics, Guru Ghasidas Vishwavidyalaya, Bilaspur (C.G.), India

In 1980, H. Izumi introduced the concept of $h$-vector. In this paper, we discuss the Finsler spaces $(M^n, L)$ and $(M^n, {}^*L)$, where ${}^*L(x, y)$ is obtained from $L(x, y)$ by Kropina change ${}^*L(x, y) = \frac{L^2(x,y)}{b_i(x,y)\, y^i}$ and $b_i(x, y)$ is an $h$-vector in $(M^n, L)$. We also find the necessary and sufficient condition when the Cartan connection coefficients for both spaces $(M^n, L)$ and $(M^n, {}^*L)$ are same.
**Keywords:** Finsler space, Kropina change, $h$-vector.
2000 Mathematics Subject Classification: **53B40**.

## 12. Title: On The Projective Flatness of a Finsler Space with arctan $(\alpha, \beta)$-metric

Gauree Shankar
Banasthali University, Banasthali

**Abstract:** Finsler projective geometry is an important part of Finsler geometry and has been studied by so many geometers, e.g., Matsumoto, Shen etc. For two Finsler spaces $(M, F)$ and $(M, \bar{F})$ on a common underlying manifold $M$, we say $(M, F)$ is projective to $(M, \bar{F})$ (or Finsler metric $F$ is projectively related to Finsler metric $\bar{F}$ ) if any geodesic of $(M, F)$ coincides with a geodesic of $(M, \bar{F})$ as a set of points and vice versa. A Finsler space $(M, F)$ is called projectively flat, if it has a covering by coordinate neighbourhoods in which it is projective to a locally Minkowski space (or Finsler metric $F$ is projectively related to a locally Minkowski metric). In fact, a projectively flat Finsler space is a Finsler space with rectilinear extremals, which was originally suggested by Hilbert. In this paper we propose to study the projectively flatness of a Finsler space with arctan $(\alpha, \beta)$-metric $F = \alpha + \beta + \beta \arctan \frac{\beta}{\alpha}$ .

13. **Title: Strictly convex functions and isometry groups of complete Finsler manifolds**

Bankteshwar Tiwari
DST-CIMS,Faculty of Science,
Banaras Hindu University, Varanasi-221005

**Abstract:** A well known classical theorem due to E. Cartan states that every compact isometry group on an Hadamard manifold has a fixed point. Yamaguchi extended this theorem to Riemannian setting and proved that if a Riemannian manifold $M$ admits a strictly convex function with minimum, then each compact subgroup of isometry group of $M$ has a common fixed point. Cartan's theorem follows from the fact that the distance function to every point on $H$ is strictly convex. In this talk Yamaguchi result is being generalized to a Finsler manifolds (joint work with Professor Shiohama).